

Fresno Math Circle

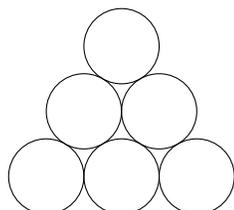
Preview Problems

Grades 10-12

Name: _____

- The first five problems are similar to some of those we do in our Math Circle meetings. In the high school grades we learn advanced math concepts and problem solving strategies and do a lot of problems from math competitions. The last three problems are from Problem Solving Competitions for high school students held at Fresno State.
- Spend as much time as needed on these problems. Do not worry if you do not solve all of the problems. These problems are challenging and are meant for you to see if you enjoy the problems we do at the Fresno Math Circle. However, please do try your best.
- For each problem, explain how you solved it (and show your calculations), and write your answer in the answer box. Please provide good and clear explanations in full sentences. We would like to see your reasoning, not just a correct answer.
- Have fun! If you enjoy solving problems and puzzles like these, you will definitely enjoy participating in the Fresno Math Circle.
- Please scan your solutions and send them to fresnomathcircle@gmail.com within one week of filling out the application form. Your work will be reviewed along with the application form.

1. In the following figure, three disks are to be colored blue, two are to be colored red, and one is to be colored green. Diagrams that can be obtained from one another by a rotation or a reflection of the whole figure are considered the same. How many different diagrams are possible?



Answer:

2. Two hikers started at dawn, one from A to B and the other from B to A . Each maintained his own constant speed for the whole hike. They met at noon (they took the same trail), and continued each his way. The first one arrived at B at 4 PM and the other one arrived at A at 9 PM. What time was dawn that morning?

Answer:

3. A jar contains $2n$ balls, n black and n white. If n people draw balls from the jar, each person drawing two random balls, then what is the probability that each person draws balls of different colors?

Answer:

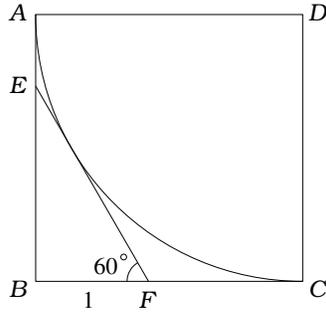
4. When all like terms in the expansion of

$$(x + y + z)^{100} + (x - y - z)^{100}$$

are collected, what is the sum of the coefficients in all of the terms?

Answer:

5. A circular arc is drawn in a square with center D at one of the vertices of the square and the arc is tangent to the opposite two sides of the square. The arc is also tangent to the hypotenuse of the $30^\circ - 60^\circ - 90^\circ$ triangle BFE as shown, where $BF = 1$. What is the radius of the circle?



Answer:

6. A rising number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. When all five-digit rising numbers are arranged from smallest to largest, find the 100th number in the list.

Answer:

7. Does there exist a triangle whose heights have lengths 1 , $\sqrt{5}$, $1 + \sqrt{5}$?

Answer: