

Fresno Math Circle

Preview Problems

8th grade

Name: _____

- Here are a few problems that are similar to some of those we frequently do in our meetings. We also learn quite a bit of advanced math concepts and problem solving strategies, play various math games, and do fun group activities.
- Spend as much time as needed on these problems. Do not worry if you do not solve all problems. These problems are challenging and are meant for you to see if you enjoy the problems we do at the Fresno Math Circle. However, please do try your best.
- For each problem, explain how you solved it (and show your calculations), and write your answer in the answer box. Please provide good and clear explanations in full sentences. We would like to see your reasoning, not just a correct answer.
- Have fun! If you enjoy solving problems and puzzles like these, you will definitely enjoy participating in the Fresno Math Circle.
- Please scan your solutions and send them to fresnomathcircle@gmail.com within one week of filling out the application form. Your work will be reviewed along with the application form.

1. There is an island far, far away, whose inhabitants are quite unusual. Some of them never tell a lie, and others never say a true statement. Those who never lie call themselves knights, and those who never tell the truth are known as knaves. Suppose you meet three inhabitants of this island, Alice, Bob, and Crystal. They make the following statements:

Alice: "We are all knaves."

Bob: "Crystal is a knight."

Crystal: "Bob is a knight."

Determine who is what.

Answer:

2. If you multiplied one hundred thirty-five 3's together (i.e. $3 \times 3 \times 3 \times \cdots \times 3$, with one hundred thirty-five 3's), what would be the ones digit of the product?

Answer:

3. Which is larger:

$$A = 99 \cdot (1 + 2 + 3 + \cdots + 100), \text{ or}$$

$$B = 100 \cdot (1 + 2 + 3 + \cdots + 99)?$$

Answer:

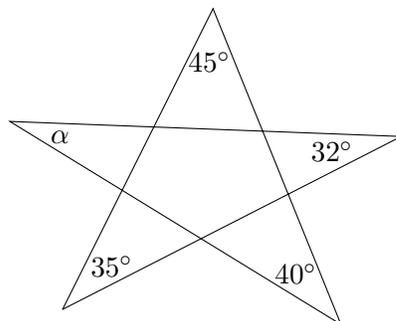
4. A train traveling at a steady speed crossed a 200 m long bridge in 1 minute. (That is, from the moment when the first car of the train entered the bridge to the moment when the last car completely left the bridge 1 minute passed.) The whole train passed a person standing on the bridge in 40 seconds. How long is the train?

Answer:

5. How many four-digit numbers have at least two equal digits?

Answer:

6. In the figure below, some angles are given. Find the degree measure of angle α .



Answer:

7. Fill in the grid so that each row and each column contains the numbers 1, 2, 3, 4, 5, and 6 once each, and the product of the numbers in any bold-lined region is as indicated. Numbers may repeat within the bold-lined regions. For this puzzle you do not have to explain how you determined all entries, just fill in the grid.

2	30	24		6	
		12		5	
24		6		20	12
15	3				
	10		30	18	4
8					