Department of Mathematics
Eighth Annual
High School Problem Solving Competition
November 1, 2023

1. **10 points** Suppose that a, b, c, d, and e are real numbers such that

$$a + b + c + d + e = 8$$

and

$$a^2 + b^2 + c^2 + d^2 + e^2 = 16.$$

Determine the maximum possible value of e.

2. **10 points** Let q be an odd integer greater than 1. Show that there is a positive integer n such that $2^n - 1$ is a multiple of q.

3. **10 points** The height of a trapezoid $ABCD$ whose diagonals AC and BD are perpendicular is equal to 4. Find the area of the trapezoid if it is known that the length of one of its diagonals AC is equal to 5.

4. **10 points** Suppose that one removes two opposite corner squares of an 8×8 chessboard. Can the remaining 62 squares be covered with 31 dominoes (of size 2×1)? Remember to justify your answer.

5. **10 points** If n^n has 10^{100} decimal digits, how many digits does n have?

6. **10 points** Prove that there do not exist (strictly) positive integers x, y, z such that

$$x^n + y^n = z^n$$

for all integers $n \geq 3$.