Department of Mathematics
Sixth Annual
High School Problem Solving Competition
October 27, 2021

1. **10 points** Let a and b be integers and $\sqrt[3]{a+6x} = b$. If $a - 6 = b$, prove that x is an integer.

2. **10 points** Find all real values of x satisfying the following equation:

$$\frac{36^x}{27^x + 64^x} + \frac{48^x}{27^x + 64^x} = \frac{1}{2}.$$

3. **10 points** Cars cross the starting line of a car race simultaneously. They arrive to the finish line one after the other with equal times elapsing among the arrival of any two cars. The average speed of the first place winner car was v_1, and the average speed of the fourth place winner car was v_4 during the race. Express the average speed of the second place winner in terms of v_1 and v_4.

4. **10 points** Alice and Bob have unfair coins. Alice's coin lands on heads with probability 2/3 and on tails with probability 1/3. She flips her coin three times and records the sequence of outcomes: H for heads and T for tails. For example, if she gets heads on the first two flips and tails on the last flip, she will record HHT. Bob's coin lands on heads with probability 1/3 and on tails with probability 2/3. He also flips his coin three times and records the sequence of outcomes. What is the probability that Alice's sequence comes before Bob's in alphabetical order?

5. **10 points** Prove that for every integer $n > 2$,

$$(1 \cdot 2 \cdot 3 \cdot \cdots \cdot n)^2 > n^n.$$

6. **10 points** In the rectangle $ABCD$, $AB = 7$ m, $AD = 4$ m. The lines AF and EC are parallel, and the distance between lines AF and EC is 1 m. Calculate the exact area of the quadrilateral $AECF$.