

Department of Mathematics
Third Annual
Adnan H. Sabuwala Problem Solving Competition
November 12, 2021

1. **10 points**

When a natural number N is written in base 5, it has three digits. When the same number is written in base 6, it has the same three digits but in the reverse order. Find all possible values of N .

2. **10 points**

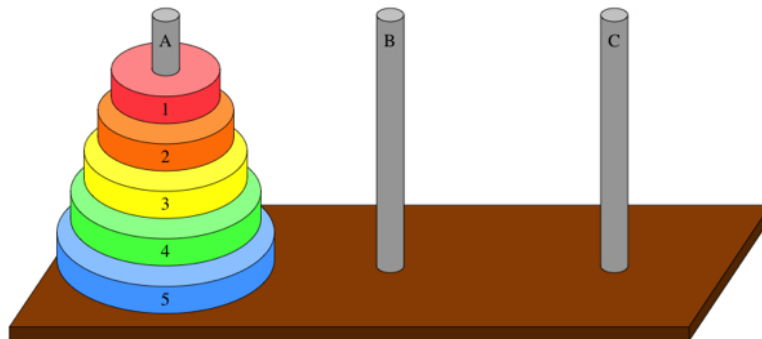
For what values of c is there a straight line that intersects the curve

$$y = x^4 + cx^3 + 12x^2 - 5x + 2$$

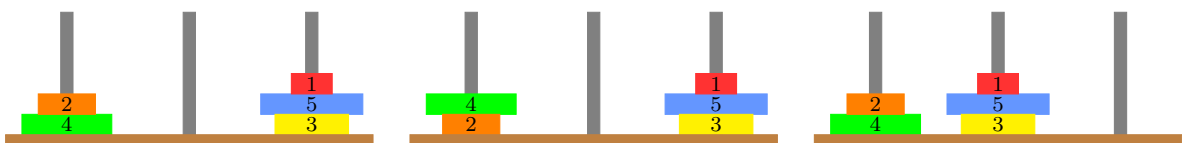
in four distinct points?

3. **10 points**

Consider the Tower of Hanoi puzzle with three rods and five different disks.



Unlike in the actual puzzle, in this problem we will allow the disks to be placed on a rod in any order (not necessarily in the decreasing order of their radii). The order of the disks matters. For example, the arrangements shown below are considered all different.



How many arrangements are possible?

4. **10 points**

Each of the values of a , b , c , and d are chosen independently and randomly from the set $\{x \in \mathbb{Z} \mid -5 \leq x \leq 5\}$. Let $\vec{v} = (a, b)$, $\vec{u} = (c, d)$, and $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Let P_1 be the probability that $\vec{v} \cdot \vec{u} = 0$. Let P_2 be the probability that $\det(M) = 0$. What is $\frac{P_1}{P_2}$?

5. **10 points**

Let $f : [0, \infty) \rightarrow (0, \infty)$ be a continuous function and a sequence $\{x_n\}_{n=1}^{\infty}$ be recursively defined as follows:

$$x_1 = 1,$$
$$x_{n+1} = x_n \left(\frac{1}{2} + \sqrt{\frac{1}{4} + f(x_n)} \right) \text{ for all } n \in \mathbb{N}.$$

Prove that

$$\lim_{n \rightarrow \infty} x_n = \infty.$$

6. **10 points**

How many binary strings (i.e., strings whose terms are zeros and ones) of length 10 have at least one instance of three consecutive zeros but no instances of four consecutive zeros? For example, strings 1100010101 and 0001110001 should be counted, but 1100000110 should not.