Problem of the Month
October 2009

Here is the problem for October 2009. You will have until the last date of the month to solve the problem. Solutions can be either

1. written neatly on a sheet of paper and turned in to me or Dr. Oscar Vega at my (PB 347) or his (PB 352) office (simply slide through the door or put it in the white mailbox outside Dr. Vega's office), or

2. typed up using your favorite text editing software (LaTeX preferred) and then turned in to me or him via email at asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the month, we will review all the turned in solutions and post the names of the individuals who have turned in complete correct solutions.

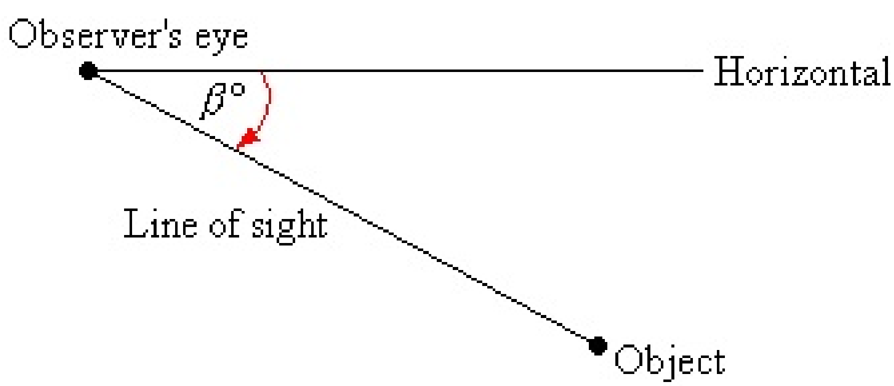
Problem for October 2009

A person on the summit of a mountain observes that the angles of depression of a car moving on a straight road (which does not pass through the foot of the mountain) at three consecutive mile markers are α, β and γ respectively. Prove that the height of the mountain is

$$\left[\frac{2}{\cot^2\alpha - 2\cot^2\beta + \cot^2\gamma}\right]^{1/2}$$

<u>Definition:</u> The angle of depression is defined to be the angle made between the horizontal and the straight line of sight of an observer looking at an object that is at a lower height than the observer.

For example, β is the angle of depression in the picture below.



Problem of the three weeks
November 2009

We are pleased to inform that Tim Courrejou wrote the best* solution for the October 2009 Problem of the Month, and that we have decided to give Michael Watkins a honorable mention for his solution. They have won the right to brag, and be correct in any mathematical discussion** that is held in November 2009. Congratulations Tim and Michael!

We thank to all those who wrote solutions to last month's problem. We encourage you to keep submitting your solutions.

You will have until **November** 24^{th} to solve the problem below (note the short 'month'). Solutions can be either

1. written neatly on a sheet of paper and dropped in the mailbox outside PB 352, or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in via email at either asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the month, we will review all your solutions and post the names of the individuals who have turned in complete correct solutions, and who wrote the best* solution.

Bragging rights winners, solutions, past (and future) problems of the month, etc can be found on

$$http://csufresno.edu/math/news\_and\_events/pom.shtml$$

Problem for November 2009.

It is clear that any number such as

$$\begin{aligned} 333 &= 3(111) \\ 555555555 &= 5(111111111) \\ 4444 &= 4(1111) \\ 77777777777777777777777 &= 7(11111111111111111111111) \end{aligned}$$

cannot be prime, however 11 is prime, so a number formed by n consecutive 1's could be prime!

Show that if

$$111111111111 \cdots 111111111111$$

(n ones in a row) is prime then n must be prime.

* A solution will be considered better than other in terms of being correct, thoroughness of the explanation, beauty of the idea used, etc.

** Exams, quizzes, homework not included. Not valid where voided and with non-participating professors.

Problem of the three weeks
December 2009

We are pleased to inform that Michael Watkins wrote the best* solution for the November 2009 Problem of the three weeks. He has won the right to brag, and be correct in any mathematical discussion** that is held in December 2009. Congratulations Michael!

We thank to all those who wrote solutions to last month's problem. We encourage you to keep submitting your solutions.

You will have until **December** 18^{th} to solve the problem below (note the short 'month'). Solutions can be either

1. written neatly on a sheet of paper and dropped in the mailbox outside PB 352, or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in via email at either asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the month, we will review all your solutions and post the names of the individuals who have turned in complete correct solutions, and who wrote the best* solution.

Bragging rights winners, solutions, future problems of the month, etc can be found on

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Problem for December 2009.
Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then prove that

$$\frac{b^2}{q^2} = \frac{ac}{pr}.$$

$*$ A solution will be considered better than other in terms of being correct, thoroughness of the explanation, beauty of the idea used, etc.

$**$ Exams, quizzes, homework not included. Not valid where voided and with non-participating professors.

Problem of the month
February 2010

We are pleased to inform that Michael Watkins wrote the best* solution for the December 2009 Problem of the Month. He has won the right to brag, and be correct in any mathematical discussion** that is held in February 2010. Congratulations Michael!

We thank to all those who wrote solutions to December's problem. We encourage you to keep submitting your solutions.

You will have until **February** 26^{th} to solve the problem below. Solutions can be either

1. written neatly on a sheet of paper and dropped in the mailbox outside PB 352, or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in via email at either asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the month, we will review all your solutions and post the names of the individuals who have turned in complete correct solutions, and who wrote the best* solution.

Bragging rights winners, solutions, past (and future) problems of the month, etc can be found on

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Problem for February 2010.

How many positive integers less than $1,000,000$ can be written as sums of at least two distinct powers of 5 (only positive exponents are to be considered)?

Generalize your solution to positive integers less than some (positive) integer N.

* A solution will be considered better than other in terms of being correct, thoroughness of the explanation, beauty of the idea used, etc.

** Exams, quizzes, homework not included. Not valid where voided and with non-participating professors.

Problem of the month
March 2010

We are pleased to inform that Michael Watkins wrote the best* solution for the February 2010 Problem of the Month. He has won the right to brag, and be correct in any mathematical discussion** that is held in February 2010. Congratulations Michael!

Tim Courrejou also wrote a wonderful numerical solution to this problem and deserves an honorable mention.

We thank to all those who wrote solutions to last month's problem. We encourage you to keep submitting your solutions.

You will have until **March** 31^{st} to solve the problem below. Solutions can be either

1. written neatly on a sheet of paper and dropped in the mailbox outside PB 352, or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in via email at either asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the month, we will review all your solutions and post the names of the individuals who have turned in complete correct solutions, and who wrote the best* solution.

Bragging rights winners, solutions, future problems of the month, etc can be found on

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Problem for March 2010.
Find the coefficient of x^{50} in the expression:

$$(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \ldots + 1000x^{999}(1+x) + 1001x^{1000}.$$

* A solution will be considered better than other in terms of being correct, thoroughness of the explanation, beauty of the idea used, etc.

** Exams, quizzes, homework not included. Not valid where voided and with non-participating professors.

Problem of the month
April 2010

We are pleased to inform that Michael Watkins wrote the best* solution for the March 2010 Problem of the Month. He has won the right to brag, and be correct in any mathematical discussion** that is held in April 2010. Congratulations Michael!

We thank to all those who wrote solutions to March's problem. We encourage you to keep submitting your solutions.

You will have until **April** 30^{th} to solve the problem below. Solutions can be either

1. written neatly on a sheet of paper and dropped in the mailbox outside PB 352, or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in via email at either asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the month, we will review all your solutions and post the names of the individuals who have turned in complete correct solutions, and who wrote the best* solution.

Bragging rights winners, solutions, past (and future) problems of the month, etc can be found on

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Problem for April 2010.

Is it possible to draw a line on $\mathbb{R}^2$ that passes through exactly one point of the form (x, y) with $x, y \in \mathbb{Z}$?

* A solution will be considered better than other in terms of being correct, thoroughness of the explanation, beauty of the idea used, etc.

** Exams, quizzes, homework not included. Not valid where voided and with non-participating professors.

Problem of the three weeks
May 2010

We are pleased to inform that Michael Watkins and James Tipton wrote the best* solution for the April 2010 Problem of the Month. They have won the right to brag, and be correct in any mathematical discussion** that is held in May 2010. Congratulations Michael and James!
Colin Aitken from Leland High School, San Jose also deserves an honorable mention for submitting a correct solution to April's problem. Congratulations Colin!

We thank to all those who wrote solutions to April's problem. We encourage you to keep submitting your solutions.

You will have until **May** 21^{st} to solve the problem below. Solutions can be either

1. written neatly on a sheet of paper and dropped in the mailbox outside PB 352, or
2. typed up using your favorite text editing software (LaTeX preferred) and then turned in via email at either asabuwala@csufresno.edu or ovega@csufresno.edu.

At the end of the three weeks, we will review all your solutions and post the names of the individuals who have turned in complete correct solutions, and who wrote the best* solution.

Bragging rights winners, solutions, past (and future) problems of the month, etc can be found on

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Problem for May 2010.

Evaluate (with proof) $\displaystyle\int_0^1 \frac{\log(1+x)}{1+x^2}\ dx.$

$*$ A solution will be considered better than other in terms of being correct, thoroughness of the explanation, beauty of the idea used, etc.

$**$ Exams, quizzes, homework not included. Not valid where voided and with non-participating professors.