

# CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10  
PART I

April 18<sup>th</sup>, 2015

1. Evaluate the following:

$$(8 - 2) \div 1 + 1 \times 4 \times 2 - 6.$$

- (a) 50
- (b) 18
- (c) 8
- (d) -10

2. If 20% of 30% of 50% of  $x$  is 30, what is  $x$ ?

- (a) 1100
- (b) 1000
- (c) 950
- (d) 900

3. If  $n$  and  $p$  are integers such that  $p > n > 0$  and  $p^2 - n^2 = 12$ , which of the following could be a possible value(s) of  $p - n$ ?

I. 1

II. 2

III. 4

(a) I. only

(b) II. only

(c) I. and II. only

(d) I., II., and III.

4. If the average of  $t$  and  $y$  is 15, and the average of  $x$  and  $w$  is 15, what is the average of  $t$ ,  $w$ ,  $x$ , and  $y$ ?

- (a) 30
- (b) 22.5
- (c) 15
- (d) 7.5

5. Consider the following list of seven numbers:

10, 18, 4, 15, 3, 21,  $x$ .

If  $x$  is the median of the number listed above, which of the following could be a value of  $x$ ?

- (a) 4
- (b) 8
- (c) 14
- (d) 16

6. The ratio of the number of men to the number of women on a committee of 20 members is 3 to 2. How many members of the committee are women?

- (a) 8
- (b) 6
- (c) 10
- (d) 12

7. Simplify the expression

$$\left(\frac{3x}{y}\right)^2 \div \left(\frac{1}{y}\right)^5.$$

(a)  $9x^2y^7$

(b)  $\frac{9x^2}{y^7}$

(c)  $\frac{9x^2}{y^3}$

(d)  $9x^2y^3$

8. What is the measure of each interior angle of a regular decagon?

- (a)  $36^\circ$
- (b)  $144^\circ$
- (c)  $360^\circ$
- (d)  $1440^\circ$

9. Determine the coefficient of  $x^2$  in  $(x - y)^{15}$ .

- (a) -1365
- (b) -105
- (c) 105
- (d) 1365

10. Triangles  $PQR$  and  $XYZ$  are similar. If  $PQ = 6$ ,  $PR = 4$ , and  $XY = 9$ , what is  $XZ$ ?

- (a) 6
- (b) 8
- (c) 12
- (d) 18

11. In how many different ways can the letters in the word STUDY be ordered?

- (a) 120
- (b) 60
- (c) 24
- (d) 2

12. If the ratio of  $2x$  to  $5y$  is 3 to 4, what is the ratio of  $x$  to  $y$ ?

- (a) 3 to 10
- (b) 8 to 15
- (c) 10 to 3
- (d) 15 to 8

13. Let the function  $f$  be defined by  $f(x) = x^2 - 7x + 10$ . If  $f(t + 1) = 0$ , which of the following could be  $t$ ?

(a)  $t = 5$

(b)  $t = 3$

(c)  $t = 1$

(d)  $t = 0$

14. How many real solutions does  $x^5 + 2x^3 + 8x^2 + 16 = 0$  have?

- (a) 0
- (b) 1
- (c) 3
- (d) 5

15. If  $x > y > 0$  and  $z \neq 0$ , the inequality which is NOT always correct is

(a)  $x - z > y - z$

(b)  $xz > yz$

(c)  $\frac{x}{z^2} > \frac{y}{z^2}$

(d)  $xz^2 > yz^2$

16. If  $3^3 \times 9^{12} = 3^x$ , what is the value of  $x$ ?

- (a) 3
- (b) 15
- (c) 21
- (d) 27

17. The odometer of my old car functions improperly, and it registers only 2 miles for every 3 miles driven. If the odometer indicates that I have driven 48 miles, how many miles has my car been driven?

- (a) 144
- (b) 96
- (c) 72
- (d) 64

18. If  $n$  is a two-digit number that can be expressed as the product of two consecutive even integers, which is NOT a possible value of  $n$ ?

- (a) 24
- (b) 48
- (c) 64
- (d) 80

19. The population of a town in California doubles every 10 years. The population in the year  $X + 100$  will be how many times the population  $P$  in year  $X$ ?

- (a)  $2^{10}P$
- (b)  $2^{20}P$
- (c)  $2^{50}P$
- (d)  $2^{100}P$

20. One sheet of metal can be melted down to make a spherical ball with a radius of 2 cm. How many such sheets would need to be melted down to make a spherical ball of radius 6 cm?

- (a) 3
- (b) 9
- (c) 16
- (d) 27

21. If  $n$  is an integer such that

$$5 < \sqrt{n} < 9,$$

what is the sum of the largest possible value of  $n$  and the smallest possible value of  $n$ ?

- (a) 2
- (b) 4
- (c) 100
- (d) 106

22. If the perimeter of a rectangle of length  $l$  and width  $w$  is equal to 24 and  $w = \frac{2}{3}l$ , what is  $w$ ?

- (a) 4.8
- (b) 7.2
- (c) 8
- (d) 9.6

23. If  $x + 2y = 20$ ,  $y + 2z = 9$ , and  $2x + z = 22$ , what is the value of  $x + y + z$ ?

- (a) 10
- (b) 12
- (c) 17
- (d) 22

24. Suppose that  $x$  and  $y$  are real numbers such that  $x^2 + y^2 = 5$  and  $xy = 7$ . What is  $x + y$ ?

- (a) 2
- (b)  $\sqrt{12}$
- (c)  $\sqrt{19}$
- (d) 12

25. Find the smallest integer  $n$  such that  $|n - 2015| < 25$ .

- (a) 1990
- (b) 1991
- (c) 2039
- (d) 2040

26. Which of the following is an equation of the perpendicular bisector of the line segment with endpoints  $(3, 4)$  and  $(15, 20)$ ?

(a)  $y = -\frac{3}{4}(x - 25)$

(b)  $y = \frac{3}{4}(x + 7)$

(c)  $y = \frac{4}{3}x$

(d)  $y = -\frac{4}{3}x + 24$

27. Bianca, Tom, and Maria are the only applicants for a job at a local cannery. Bianca and Tom are equally likely to get the job, but Maria's chance of being hired is 20% higher than Tom's due to her superior work skills. What is the probability that Maria will be hired?

- (a) 0.625
- (b) 0.2
- (c) 0.3125
- (d) 0.375

28. Three balls are stacked in a cylinder that touches the stack on all sides and on the top and bottom. What is the ratio of the volume of balls to the volume of the cylinder?

(a)  $\frac{2}{9}$

(b)  $\frac{2}{3}$

(c)  $\frac{4}{9}$

(d)  $\frac{4}{3}$

29. If  $f(g(x)) = x^4$  and  $f(x) = (x + 1)^2$ , which of the following could be  $g(x)$ ?

(a)  $x^2 - 1$

(b)  $x^2 + 1$

(c)  $x^4 - 1$

(d)  $\frac{x^4}{(x + 1)^2}$

30. Suppose  $p(x)$  is a polynomial of degree 2, has roots 3 and -5, and  $p(0) = -60$ . What is the coefficient of  $x$  in  $p(x)$ ?

- (a) 1
- (b) 6
- (c) 7
- (d) 8



## Solutions

1 C

2 B

3 B

4 C

5 C

6 A

7 D

8 B

9 B

10 A

11 A

12 D

13 C

14 B

15 B

16 D

- 17 C
- 18 C
- 19 A
- 20 D
- 21 D
- 22 A
- 23 C
- 24 C
- 25 B
- 26 A
- 27 D
- 28 B
- 29 A
- 30 D

# CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10  
PART II

April 18<sup>th</sup>, 2015

1. What is the remainder when  $x^{100} - 2x^{51} + 1$  is divided by  $x^2 - 1$ ?

- (a)  $2x + 2$
- (b)  $2x - 2$
- (c)  $-2x + 2$
- (d)  $-2x - 2$

2. A fair die is rolled six times. What is the probability of rolling five or six at least five times?

(a)  $\frac{12}{729^2}$

(b)  $\frac{1}{729}$

(c)  $\frac{12}{729}$

(d)  $\frac{13}{729}$

3. Suppose that you row upstream at 3 miles per hour and you return on the same route at 6 miles per hour. What was your average speed (in miles per hour) for the entire trip?

- (a) 4
- (b) 4.5
- (c) 5
- (d) 5.5

4. Suppose  $S$  is a set of positive integers, each of which is less than 25, such that no two elements of  $S$  have a common divisor greater than 1. What is the largest possible number of elements in  $S$ ?

- (a) 3
- (b) 6
- (c) 8
- (d) 10

5. Let  $\overline{BE}$  be a median of triangle  $ABC$ , and let  $D$  be a point on  $\overline{AB}$  such that  $\frac{BD}{DA} = \frac{3}{7}$ . What is the ratio of the area of triangle  $BED$  to that of triangle  $ABC$ ?

(a)  $\frac{3}{10}$

(b)  $\frac{10}{3}$

(c)  $\frac{3}{20}$

(d)  $\frac{10}{6}$

6. The recipe for limeade calls for 4 ounces of lime juice for every 12 ounces of water. Cristina initially uses 18 ounces of water to make her limeade. If her limeade has 40% too much lime juice, how many ounces of water does she need to add to her mixture to have the correct ratio of lime juice to water?

- (a) 6 ounces
- (b) 7.2 ounces
- (c) 25.2 ounces
- (d) 45 ounces

7. During the winter, 60% of college students ski, and during the summer, 45% of college students hike. If 15% of college students do both activities, what percent of college students do neither?

- (a) 10%
- (b) 15%
- (c) 85%
- (d) 90%

8. A candy store sells bags of 40 caramels for \$3.20, bags of 40 chocolates for \$4.00, and mixed bags of chocolates and caramels for \$3.50. If the mixed bags also have 40 pieces of candy, how many caramels are in each mixed bag?

- (a) 15
- (b) 20
- (c) 25
- (d) 30

9. If  $\frac{1}{x} + \frac{1}{y} = 5$  and  $\frac{1}{x} - \frac{1}{y} = 1$ , what is  $x + y$ ?

(a)  $\frac{1}{4}$

(b)  $\frac{1}{3}$

(c)  $\frac{1}{2}$

(d)  $\frac{5}{6}$

10. A square and a triangle have equal perimeters. If the lengths of the three sides of the triangle are 5.2, 6.1, and 8.7, what is the area of the square?

- (a) 25
- (b) 20
- (c) 16
- (d) 5

11. In how many ways can six people be seated in a circle, if two arrangements are considered the same whenever each person has the same neighbors (not necessarily on the same side)?

- (a) 20
- (b) 60
- (c) 120
- (d) 360

12. Three identical coins of radius 1 are placed on a table so that they are mutually tangent. A smaller coin is placed between them tangent to all 3. What is the radius of the smaller coin?

(a)  $\frac{1}{3}$

(b)  $\frac{2}{\sqrt{3}} - 1$

(c)  $\sqrt{2} - 1$

(d)  $\frac{1}{2\sqrt{3}}$

13. A paper cone has height 12 inches and the diameter of the base has length 10 inches. The cone is cut along one side and unrolled to form a portion of a disk. What angle of the circle does this portion include?

(a)  $\frac{5\pi}{13}$

(b)  $\frac{5\pi}{12}$

(c)  $\frac{10\pi}{13}$

(d)  $\frac{5\pi}{6}$

14. A number is a palindrome if it reads the same forward as backwards, such as 10201. How many integers between 10000 and 99999 (inclusive) are palindromes?

- (a) 729
- (b) 810
- (c) 900
- (d) 999

15. Let  $a$  and  $b$  be integers whose sum  $a + b$  is divisible by three. Which of the following must be true?

- I.  $a^2 + b^2$  is divisible by three.
- II.  $a^2 - b^2$  is divisible by three.
- III.  $a^3 + b^3$  is divisible by three.

- (a) I only
- (b) II only
- (c) I and III
- (d) II and III

16. Carlos can wash my car in 90 minutes. Patrick can wash my car in 2 hours. Rachel has never washed my car before but wants to help today. All three of them work together and wash my car in 36 minutes. At the rate that Rachel was going, how long would it have taken her to wash my car alone?

(a)  $\frac{16}{17}$  hours

(b)  $\frac{3}{2}$  hours

(c)  $\frac{17}{10}$  hours

(d) 2 hours

17. An alchemist knows that one jar equals one bottle plus one glass, two jars equals seven glasses, and one bottle equals one cup plus two glasses. How many cups are there in a bottle?

- (a) 5
- (b) 5
- (c) 3
- (d) 2

18. The perimeter of square of area 1 is equal to the circumference of a circle  $C$ . Which of the following numbers is closest to the area of  $C$ ?

- (a) 1
- (b) 1.1
- (c) 1.2
- (d) 1.3

19. An isosceles right triangle has legs of length  $4^{10}$ . How long is its hypotenuse?

(a)  $4^{10.25}$

(b)  $4^{10.5}$

(c)  $4^{10.75}$

(d)  $4^{11}$

20. Which of the following numbers is a perfect square?

(a)  $\frac{14!15!}{2}$

(b)  $\frac{15!16!}{2}$

(c)  $\frac{16!17!}{2}$

(d)  $\frac{17!18!}{2}$

21. Monica pays for four muffins and three bananas. Charles spends twice as much paying for two muffins and sixteen bananas. The price of a muffin is how many times the price of a banana?

(a)  $\frac{3}{2}$

(b)  $\frac{5}{3}$

(c)  $\frac{7}{4}$

(d) 2

22. David drives from his home to the airport to catch a flight. He drives 35 miles per hour in the first hour, but realizes that he will be 1 hour late for his flight. He increases his speed by 15 miles per hour for the rest of the trip to the airport and arrives 30 minutes early. How many miles is the airport from his home?

- (a) 140
- (b) 175
- (c) 210
- (d) 245

23. Cucumbers contain 99% water by weight. After being exposed to the sun, the amount of water drops to 98%. What percentage of weight did the cucumbers lose?

- (a) 20%
- (b) 2%
- (c) 50%
- (d) 5%

24. A leather jacket was originally priced at \$200. It was successively discounted by 10%, 15%, and 20%. What was the price of the jacket after the third discount?

- (a) \$170.00
- (b) \$122.40
- (c) \$110.00
- (d) \$90.00

25. Find the sum of the solutions of the equation

$$||x + 1| - 2| = 4.$$

- (a) -2
- (b) 0
- (c) 2
- (d) 12

26. What is the sum of the infinite series

$$0.3 + 0.3^2 + 0.3^3 + 0.3^4 + \dots?$$

(a)  $\frac{107}{250}$

(b)  $\frac{3}{11}$

(c)  $\frac{3}{7}$

(d)  $\frac{7}{10}$

27. A cylinder's radius is increased by 20% and its height is decreased by 20%. How has its volume changed?

- (a) Increased by 15.2%
- (b) Decreased by 71.2%
- (c) Increased by 28.8%
- (d) Decreased by 84.8%

28. Suppose you have an equal number of pennies, nickels, and quarters in your hand. The total amount of money in your hand is \$4.65. How many total coins do you have in your hand?

- (a) 10
- (b) 15
- (c) 30
- (d) 45

29. One number is eight less than three times another number. The sum of the numbers is thirty-six. What is the larger number?

- (a) 25
- (b) 31
- (c) 50
- (d) 60

30. The diagonal of a square is  $3\sqrt{2}$  inches in length. What is the area of the square?

- (a)  $6\sqrt{2}$  square inches
- (b) 9 square inches
- (c)  $9\sqrt{2}$  square inches
- (d) 12 square inches



## Solutions

1 C

2 D

3 A

4 D

5 C

6 B

7 A

8 C

9 D

10 A

11 B

12 B

13 C

14 C

15 D

16 D

- 17 A
- 18 D
- 19 A
- 20 D
- 21 B
- 22 C
- 23 C
- 24 B
- 25 A
- 26 C
- 27 A
- 28 D
- 29 A
- 30 B