

Math Field Day
2008
Leap Frog, grades 6-8

Exam I

1. Find the original price of a pair of shoes if the sale price is \$102 after a 25% discount.

- A. \$25.50
- B. \$76.50
- C. \$204
- D. \$136
- E. \$102.25

Solution: (D)

$$0.75x = 102 \text{ or equivalently; } x - .25x = 102, \quad x = \$136$$

2. A company wants to make a trail mix that costs \$2.50 a pound. How many pounds of raisins that cost \$3 a pound must be added to 20 pounds of peanuts that cost \$2 a pound and 5 pounds of candy pieces that cost \$4 a pound?

- A. 7.5
- B. 5
- C. 28
- D. 3
- E. 13

Solution: (B)

20 pounds of peanuts and 5 pounds of candy pieces totals 25 pounds, and costs \$60.

$$\frac{60 + 3x}{25 + x} = 2.5$$

$$62.5 + 2.5x = 60 + 3x$$

$$2.5 = 0.5x$$

$$x = 5lbs$$

3. How many times does the number 7 occur in the digits of the natural numbers less than 100?

- A. 7
- B. 10
- C. 19
- D. 20
- E. 21

Solution: (D)

7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 77 (occurring twice), 78, 79, 87, 97.

4. One-square-foot floor tiles come 24 to a package. Find out how many packages are needed to cover a rectangular floor 18 feet by 21 feet.

- A. 378
- B. 9072
- C. $15\frac{3}{4}$
- D. 24
- E. $13\frac{2}{3}$

Solution: (C)

$$18 \times 21 = 378 \text{ ft}^2$$

$$\frac{378}{24} = 15.75 \text{ packages}$$

5. Twice the difference of a number and 3 is the same as 1 added to three times the number. Find the number.

- A. -3
- B. 2
- C. -4
- D. 5
- E. -7

Solution: (E)

$$2(x - 3) = 1 + 3x$$

$$2x - 6 = 1 + 3x$$

$$x = -7$$

6. A group of horses and ducks has 148 legs and 60 heads. How many of each animal are in their respective group?

- A. 46 horses and 14 ducks.
- B. 13 horses and 47 ducks.
- C. 14 horses and 46 ducks.
- D. 43 horses and 17 ducks.
- E. 37 horses and 30 ducks.

Solution: (C)

Let h = # of horses and d = # of ducks; then since horses have 4 legs and ducks have two legs, and horses and ducks each have one head, then:

$$4h + 2d = 148$$

$$h + d = 60$$

$$4(60 - d) + 2d = 148$$

$$240 - 4d + 2d = 148$$

Letting $h=60-d$ and substituting, gives $240 - 2d = 148$

So, $h=60-46=14$ horses.

$$92 = 2d$$

$$d = 46 \text{ ducks}$$

7. Which number is a multiple of 9?

- A. 11111
- B. 13131
- C. 16161
- D. 19191
- E. 23282

Solution: (B)

Number fact: numbers are divisible by 9 if and only if their digits add to a multiple of 9.

8. Five dogs are placed two at a time on a balance scale. Spot is heavier than Rover, Maggie weighs more than Peabody but less than Lassie. Peabody weighs more than Rover. Lassie weighs less than Spot. List the dogs in order of their weights, with the lightest dog listed first.

- A. Lassie, Peabody, Rover, Maggie, Spot
- B. Rover, Maggie, Peabody, Spot, Lassie
- C. Rover, Peabody, Maggie, Lassie, Spot
- D. Spot, Lassie, Maggie, Peabody, Rover
- E. Maggie, Lassie, Spot, Rover, Peabody

Solution: (C); Spot>Rover and Lassie>Maggie>Peabody and Peabody>Rover and last, Spot>Lassie, thus: from lightest to heaviest: **Rover, Peabody, Maggie, Lassie, Spot.**

9. A sheep can clear a field of grass in 24 hours and a cow can clear the same field in 12 hours. How long (in hours) does it take for the combined efforts of the sheep and cow, to clear one field?

- A. 8 hours
- B. 4 hours
- C. 18 hours
- D. 5 hours
- E. 12 hours

Solution: (A); In 24 hours, the sheep clears 1 field and the cow clears 2 fields. Thus, together they can clear 3 fields/24 hours. Just inverting this gives us the answer; 8 hours/field, or **8 hours.**

10. On dry land, Sponge-Bob is 10% water and weighs a total of 2 ounces. After going to the Crusty Crab at Bikini Bottom (the ocean), he gets back on land but is now 85% water! What is Sponge-Bob's total weight now?

- A. 12 ounces
- B. 1 ounce
- C. 5 ounces
- D. 3.5 ounces
- E. 2 ounces

Solution: (A); Since he initially weighed 2 ounces, then at that initial point he was composed of 0.2 ounces of water and 1.8 ounces of sponge. Since he is now 85% water, then 15% must be sponge; so 0.15 times X must equal 1.8 ounces;

$0.15 \cdot X = 1.8$, which implies that his new total weight was **12 ounces**.

Exam II

1. A bottle and a cork together cost \$1.05. The bottle costs \$1 more than the cork. How much does the cork cost?

- A. 5 cents
- B. 10 cents
- C. 3 cents
- D. 2.5 cents

Solution: (D); Let X =cost of cork. Then $X + (1+X) = 1.05$.
 $2X + 1 = 1.05$; $2X = .05$; $X = 0.025$ or **2.5 cents**.

2. Convert the number $1.454545\dots$ into a *reduced* fraction of integers.

- A. $\frac{3}{2}$
- B. $\frac{144}{99}$
- C. $\frac{16}{11}$
- D. $\frac{1454545}{1000000}$
- E. $\frac{3.52}{2.42}$

Solution: (C); Let $X = 1.4545\dots$, then $100X = 145.45\dots$. Subtracting the two we get: $100X - X = 99X = (145 - 1) = 144$ and then solving for X gives: $X = \frac{144}{99} = \frac{16}{11}$. Note: answer E gives the same $1.4545\dots$, however, it is not a fraction of integers!

3. Consider the sequence of numbers 1,2,3,4,5,6,7,8,9,1,0,1,1,1,2,1,3,1,4,...
What is the number in the 100th position?

- A. 5
- B. 0
- C. 1
- D. 3
- E. 6

Solution: (A)

Write out the sequence a bit more in order to see a pattern:

1,2,3,4,5,6,7,8,9,1,0,1,1,1,2,1,3,1,4,1,5,1,6,1,7,1,8,1,9,2,0,2,1,2,2,2,3,2,4,2,5,2,6...

After the 0 in the 11th position, every 20th position gives a 0; So, to reach a hundred we need $11+4(20)+9$. After the fourth 20, we will have a 5 before the 0 at the 91st position. We then proceed as follows: 5,1,5,2,5,3,5,4,5. Thus, there will be a 5 at the 100th position.

4. Lenny can ride his bicycle uphill at 5 miles/hour and downhill at 20 miles/hour. How far uphill in miles should he travel if he wants his round trip (uphill plus downhill) to last 2 hours?

- A. 2 miles
- B. 12 miles
- C. 15 miles
- D. 7 miles
- E. 8 miles

Solution: (E); distance = rate X time; so time = distance/rate;

$$\frac{d}{5} + \frac{d}{20} = 2$$
$$\text{time_up} + \text{time_down} = 2 \text{ hours; } \frac{20d + 5d}{100} = 2$$
$$25d = 200$$
$$d = \frac{200}{25} = 8 \text{ miles}$$

5. Express the next term (in base 7) in the arithmetic sequence $2_3, 13_4, 22_5, 25_6, ?$

- A. 27_7
- B. 54_7
- C. 23_7
- D. 31_7
- E. 32_7

Solution: (D); Converting the numbers to a common base 10, we have: 2, 7, 12, 17, ; where we see the sequence is arithmetic, adding 5 each time. The next number in base ten is thus: 22.

22 in base 7 is: $3(7^1) + 1(7^0) = 31_7$.

6. Find four consecutive odd integers such that twice the first subtracted from the sum of the other three is fifteen.

- A. 5, 7, 9, 11

- B. 1, 3, 5, 7
- C. 3, 5, 7, 9
- D. 7, 9, 11, 13
- E. 11, 13, 15, 17

Solution: (C); Let $2n+1$, $2n+3$, $2n+5$, $2n+7$ be our 4 consecutive odd integers; now form the following equation: $(2n+3 + 2n+5 + 2n+7) - 2(2n+1) = 15$. After simplifying this expression, we are left with: $6n+15-4n-2=2n+13=15$; which simplifies to $n=1$; thus we have 3, 5, 7, 9 as the solution.

7. A jet plane traveling at 500 mph overtakes a propeller plane traveling at 200 mph that had a 2-hour head start. How far from the starting point are the two planes?

- A. 375 miles
- B. 300 miles
- C. 400 miles
- D. 498 miles
- E. 500 miles

Solution: (A); $500t = 200(t + 2)$
 $500t = 200t + 400$
 $300t = 400$ so; $d=500 \cdot \frac{4}{3} = 125 \cdot 3 = 375 \text{ miles}$
 $t = \frac{4}{3} \text{ hour}$

8. A car's gasoline tank is $\frac{1}{2}$ full. After adding 7 gallons of gas, the gauge shows that the tank is $\frac{3}{4}$ full. How many gallons does the tank hold?

- A. 16
- B. 12
- C. 28
- D. 32

Solution: (C); $\frac{1}{2}x + 7 = \frac{3}{4}x$
 $\frac{3}{4}x - \frac{1}{2}x = \frac{1}{4}x = 7$
 $x = 28 \text{ gallons}$

9. Suppose that x , y , z are positive numbers, and $(x + y + z)^x = 4$, $(x + y + z)^y = 2$, and $(x + y + z)^z = 32$. Evaluate z (as a fraction in lowest terms).

- A. $\frac{5}{2}$

- B. $\frac{7}{2}$
- C. 12
- D. 3

Solution: (A); Multiply the 3 equations together; we get $(x + y + z)^{x+y+z} = 256 = 4^4$,
so $x + y + z = 4$. Therefore $4^x = 32$; but $2^5 = 32$ and $4^{\frac{1}{2}} = 2$ or; $\left(4^{\frac{1}{2}}\right)^5 = 4^{\frac{5}{2}} = 32$ implying $z = 5/2$.

10. There are 5 suspects in a robbery: Homer, Marge, Bart, Lisa, and Maggie. Each makes a statement:

Homer: Lisa is guilty.

Marge: Homer and Bart are not both innocent.

Bart: If Homer is guilty, then so is Marge.

Lisa: If Marge is guilty, then so is Maggie.

Maggie: If Lisa is guilty, then so is Bart.

Each innocent suspect told the truth, each guilty suspect lied. Name all of the culprits.

- A. Bart and Homer
- B. Maggie and Homer
- C. Bart and Lisa
- D. Marge and Maggie
- E. Bart

Solution: (A); Maggie's statement says that Lisa is innocent or Bart is guilty, that is, either Homer is guilty or Bart is guilty. Therefore Maggie's statement is equivalent to Marge's statement, which means that Lisa is innocent. Therefore Homer is guilty, so Marge is innocent, so Bart is guilty and Maggie is innocent. The culprits are Homer and Bart.

