

2017
Leap Frog Relay Grades 9-10
Part I

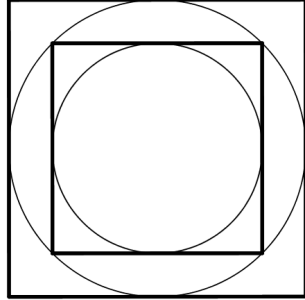
No calculators allowed

Correct Answer = 4, Incorrect Answer = -1, Blank = 0

1. How many (distinct) real number solutions are there to the equation:

$$x^{2021} - 4x^{2019} = x^{2019} - 4x^{2017}$$

- (a) 2 (b) 3
(c) 4 (d) 5
(e) None of these
2. A palindromic number is a number that is exactly the same if the order is reversed; for example, 1225221 or 1331. How many four-digit palindromic numbers are there whose sums of digits are divisible by four?
(a) 20 (b) 45
(c) 50 (d) 100
(e) None of these
3. A circle is inscribed in a square. A square is inscribed in that circle. A second circle is inscribed in that square. What is the ratio of the area of the smallest circle to the area of the largest square?



- (a) $\pi/2$ (b) $\pi^2/4$
(c) $\pi/8$ (d) $\pi^2/16$
(e) None of these

4. The graphs of $y^2 = x^6$ and $|y| = |x|$ are plotted on the Cartesian plane. How many different regions do these graphs divide the plane into?

- (a) 8 (b) 12
(c) 14 (d) 16
(e) None of these

5. An urn contains ten balls, either red or blue. You may take balls from the urn one at a time and you do not place them back into the urn. You must take at least seven balls to guarantee to take at least three red balls. How many blue balls are in the urn at the start?

- (a) 3 (b) 4
(c) 5 (d) 6
(e) None of these

6. The average wage at a local restaurant is \$10 an hour among 24 employees. A new manager is hired and the new average wage is \$10.40 an hour among the total 25 employees. How much does the new manager make per hour?
- (a) 20 (b) 22
(c) 24 (d) 26
(e) None of these
7. A parabola $y = ax^2 + bx + c$ has its vertex at $(6, 15)$ and contains the point $(0, -3)$. What is the product abc ?
- (a) $-\frac{4}{9}$ (b) -1
(c) 4 (d) 9
(e) None of these
8. The value of x is 40% of y . What is y as a percent of x ?
- (a) 60% (b) 140%
(c) 250% (d) 400%
(e) None of these
9. The sum of eight consecutive integers is 212. What is the sum of the first and last integers?
- (a) 52 (b) 53
(c) 54 (d) 55
(e) None of these

10. If $4^{5x+3} = 8^{2x+1}$, then $16^x = ?$

(a) $\frac{3}{4}$

(b) $\frac{-3}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{8}$

(e) None of these

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Part II

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11. Find the real solution to the equation:

$$\frac{1}{x} + \frac{x}{5} = \frac{1+x}{x+5}$$

(a) $x = -\sqrt[3]{25}$

(b) $x = -\sqrt[3]{10}$

(c) $x = 1 - \sqrt[3]{10}$

(d) $x = 1 - \sqrt[3]{25}$

(e) None of these

12. The lines $y = 2x + b$ and $y = x + 2017$ meet on a point on the line $y = 4x + 16$. Determine b .

(a) 1250

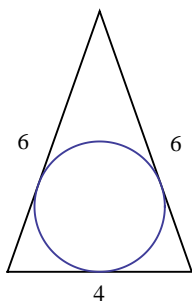
(b) 1251

(c) 1351

(d) 1350

(e) None of these

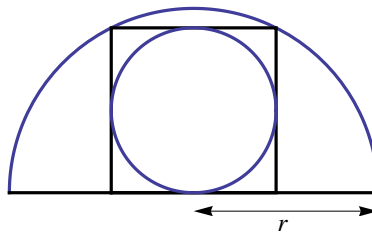
13. A circle is inscribed in the isosceles triangle with respective side lengths 6, 6 and 4. Determine the area of the inscribed circle.



- (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$
(c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$
(e) None of these
14. A music player has a list price of \$100. However, the store is having a 10% off sale for the month of April. But you are in luck, because you came on a Tuesday in April when the store gives an additional 15% *off the sale price* at the register. Assuming sales tax is 10% of the register price, how much are you going to pay for the music player?
- (a) \$83.85 (b) \$83.95
(c) \$84.05 (d) \$84.15
(e) None of these
15. $a + b + c = -4$ and $ab + ac + bc = 3$. What is $a^2 + b^2 + c^2$?
- (a) 10 (b) 16
(c) 19 (d) 25
(e) None of these
16. Suppose you take two steps to the right, four steps to the left, six steps to the right, and so on, where each number of steps is a multiple of two. Suppose the last move is 17422 steps to the right. How far are you from the starting position after the last step?

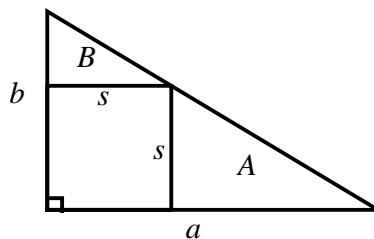
- (a) 8710 steps to the left
- (b) 8712 steps to the right
- (c) 13065 steps to the left
- (d) 17422 steps to the right
- (e) None of these

17. The semicircle pictured below has a radius of r inches. The square is inscribed in the semicircle and the smaller circle is inscribed in the square. What is the area of the smaller circle in terms of r ?



- (a) $\frac{\pi r^2}{6}$ inches².
 - (b) $\frac{\pi r^2}{5}$ inches².
 - (c) $\frac{\pi r^2}{4}$ inches².
 - (d) $\frac{\pi r^2}{3}$ inches².
 - (e) None of these
18. Four oranges weigh as much as three apples. Nine limes weight as much as two oranges. How many limes weight as much as seven apples weigh?
- (a) 21
 - (b) 28
 - (c) 35
 - (d) 42
 - (e) None of these
19. Suppose $\log_2(\log_3(\log_2 x)) = 1$. Which of the following is true about x ?
- (a) $10 < x \leq 100$
 - (b) $100 < x \leq 1000$
 - (c) $1000 < x \leq 10000$
 - (d) $x > 10000$
 - (e) None of these

20. In the figure below, the large right triangle has respective leg lengths a and b as pictured. The s by s square is inscribed in the triangle. The respective areas of the two smaller right triangles are A and B as indicated. Determine the ratio of the areas A/B as a function of a and b .



- (a) $A/B = a^2/b^2$ (b) $A/B = a/b$
(c) $A/B = (ab)/(a + b)$ (d) $A/B = \sqrt{a^2 + b^2}/(a + b)$
(e) None of these