2011
Leap Frog Relay Grades 6-8
Part I Solutions

No calculators allowed
Correct answer $= 4$**, Incorrect answer** $= -1$**, Blank** $= 0$

1. The average age of 6 people in a room is 13 years. A 3-year-old kid leaves the room. What is the average age of the five remaining people?
   (a) 10 years
   (b) 11 years
   (c) 13 years
   (d) 15 years
   (e) 16 years

   Solution. (d) Originally the sum of the ages of the 6 people in the room is $6 \cdot 13 = 78$. After the 3-year-old kid leaves, the sum of the ages of the remaining people is 78-3=75. So the average age of the five remaining people is 75/5=15.

2. What is the sum of the positive factors of 147?
   (a) 147
   (b) 148
   (c) 158
   (d) 228
   (e) 294

   Solution. (d) Since $147 = 3 \cdot 7 \cdot 7$, the positive factors of 147 are 1, 3, 7, 21, 49, 147. The sum of these factors is 228.

3. By the end of January, the Running Rabbits had won 40% of their soccer games. During February, they won 4 more games and lost only one, finishing the season having won half of their games. How many games did the Running Rabbits play in the whole season?
   (a) 10
   (b) 15
   (c) 20
   (d) 25
   (e) 30

   Solution. (c) Let n be the number of games played by the Running Rabbits until the end of January. Then $0.4n + 4 = 0.5(n + 4 + 1)$. Solving for n yields

$$\begin{aligned} 0.4n + 4 &= 0.5(n + 5) \\ 0.4n + 4 &= 0.5n + 2.5 \\ 0.4n + 1.5 &= 0.5n \\ 1.5 &= 0.1n \\ 15 &= n \end{aligned}$$

So the total number of games played in the season is $n + 4 + 1 = 15 + 4 + 1 = 20$.

4. Carla has 60 ribbons, out of which 25% are red, 30% are white, and 45% are blue. If she buys 10 more white ribbons and 5 more blue ribbons, what percentage of her ribbons will be red?
   (a) 10
   (b) 15
   (c) 20
   (d) 25
   (e) 30

   Solution. (c) Carla had $(0.25) \cdot 60 = 15$ red ribbons. If she buys 15 more ribbons (none of them were red), she will have 60+15=75 ribbons. The 15 red ribbons are $100 \cdot 15/75 = 100 \cdot 0.2 = 20$ percent of the total number of ribbons.

5. I gave a value to every vertex of a cube. The value of an edge is the sum of the values of the vertices at its ends. The value of a side is the sum of the values of the edges surrounding it. The value of a cube is the sum of the values of its sides. What is the value of the cube if the sum of the values of its vertices is 128?
   (a) 128
   (b) 256
   (c) 384
   (d) 512
   (e) 768

   Solution. (e) There are 3 edges starting from each vertex of the cube, therefore the sum of the values of the edges is 3 times the sum of the values of the vertices. Each edge is counted as a border of 2 sides, therefore the sum of the values of the sides is 2 times the sum of the values of the edges, so 6 times the sum of the values of the vertices: $6 \cdot 128 = 768$.

6. Captain Eyebrow and his brave soldiers saved the people of Funnyrunny Island from the green dragon. As a present, the captain received a basket of coconuts from the locals, which he distributed among his three soldiers fairly, but not equally. Ha-Ha received a third of what Ho-Ho got, Hey-Hey received 25 less than the two others together. What is the sum of the coconuts that Hey-Hey and Ha-Ha received if there were 95 coconuts in the basket?
   (a) 50
   (b) 55
   (c) 60
   (d) 65
   (e) 70

   Solution. (a) Let x be the number of coconuts Ho-Ho received. Then Ha-Ha got the third of x, and Hey-Hey got $x + (1/3)x - 25$ coconuts. The sum of these three quantities is 95.
   $$x + (1/3)x + x + (1/3)x - 25 = 95.$$
   Solving it for x, $x = 45$. The sum of the coconuts received by Hey-Hey and Ha-Ha together is $95 - x = 95 - 45 = 50$.

7. The ages of a father and his two different-aged sons are the powers of the same prime number. Last year everybody's age was a prime number. What is the sum of their ages this year?
   (a) 43
   (b) 44
   (c) 45
   (d) 46
   (e) 47

   Solution. (b) The ages of the 3 people are either all odd or all even, since they are the powers of the same prime number. One year ago they were also either all odd or all even, but, since 2 is the only even prime number, they all had to be odd, since they were all prime numbers. Therefore, this year the ages of each person must be even. That means they all need to be powers of 2. The powers of 2 that can be ages of any person are 2, 4, 8, 16, 32, and 64. The way each person could have prime number years a year ago is only if the sons are 4 and 8, and the father is 32 now. Therefore, the sum of their ages is 44.

8. Two bicycle clubs organize a tour together. At the meeting in the morning members greet each other with a handshake. Everybody shakes hands with everybody once. There were a total of 231 handshakes but 119 of them happened between members of the same club. How many more people came from one club than from the other?
   (a) 0
   (b) 3
   (c) 6
   (d) 8
   (e) 10

   Solution. (c) Let n be the total number of participants from the two clubs. Since every person shook hands with everybody else, $n(n-1)/2 = 231$, so $n = 22$. (We divide by 2 since a handshake is between two people). If there are x many members from one club, then $22-x$ many members are there from the other club. The number of handshakes among members of each club are $x(x-1)/2$ and $(22-x)(22-x-1)/2$. Then solving for x the equation $x(x-1)/2 + (22-x)(22-x-1)/2 = 119$ results $x = 8$ or $x = 14$. Therefore, there are 8 members from one club, and 14 members from another club present. The difference is 6 people.

9. A very thin wire is going upwards in a spiral from the bottom of a cylinder to the top, rising by the same rate all along, making 6 complete rotations around the cylinder. The radius of the cylinder is 12 centimeters, and its height is 3 meters. Which of the following quantities is the closest to the length of the wire?
   (a) 3.3 m
   (b) 4.5 m
   (c) 5.4 m
   (d) 6.3 m
   (e) 9.1 m

Solution. **(c)** If we roll out the wire, it will be the hypotenuse of a right triangle where one leg is the height of the cylinder and the other is 6 times the circumference of its circular base. If x is the length of the wire, then $x^2 = 3^2 \text{ m}^2 + (6 \cdot 2\pi \cdot 0.12)^2 \text{ m}^2$. Solving for x, we get x is about 5.4 m.

10. A cylinder-shaped coffee can have a 5 cm radius and a 7 cm height. Calculate the surface area of the can.
    (a) 80π cm^2
    (b) 90π cm^2
    (c) 100π cm^2
    (d) 110π cm^2
    (e) 120π cm^2

    Solution. **(e)** The area of the top of the can is 25π cm^2, the area of the bottom is also 25π cm^2. The area of surface connecting the top with the bottom is $2\pi \cdot 5 \cdot 7$ cm^2 $= 70\pi$ cm^2. The surface area of the can is the sum of these three quantities, namely 120π cm^2.

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Part II Solutions

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Correct answer $= 4$, **Incorrect answer** $= -1$, **Blank** $= 0$

1. The rent-a-stall pet kennel has stalls for 1000 cats and dogs. Forty percent of the stalls are for cats. On Tuesday, there were 200 cats and a bunch of dogs at the kennel. The pet kennel was 75 percent full. How many dogs were in the kennel?

   (a) 750
   (b) 1250
   (c) 1000
   (d) 100
   (e) 550

   Solution: (e)

$$\begin{aligned} 75\% \text{ full} &= \frac{75}{100} \cdot 1000 \text{ pets} \\ &= 750 \text{ pets} \end{aligned}$$

   If there are 200 cats, there must be (750 - 200) = 550 dogs.

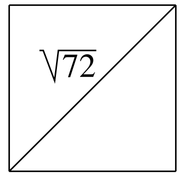
2. If a 26 mile marathon has watering stops every $3\frac{2}{5}$ miles. How many watering stops are there in the whole marathon if there is a watering stop at the beginning of the race?

   (a) 8 stops
   (b) 11 stops
   (c) 12 stops
   (d) 10 stops
   (e) 7 stops

   Solution: (a) $26 \div 3\frac{2}{5} = 26 \cdot \dfrac{5}{17} \approx 7.65$ so there are 7 stops in addition to the beginning water stop; thus, there are 8 watering stops in the whole marathon.

3. What is the perimeter of a square with a diagonal of length $\sqrt{72}$ as shown in the figure below?

   (a) $2\sqrt{72}$
   (b) 24
   (c) 12
   (d) $2\sqrt{36}$
   (e) 36



Solution: (b) By the Pythagorean theorem, $a^2 + a^2 = \sqrt{72}^2$ hence, $2a^2 = 72$, thus the square has sides of length 6. Therefore the perimeter is 24.

4. Four notorious thieves named Rickey, Lou, Maury and Ty are accused of theft at El Segundo military base. Each makes a statement.

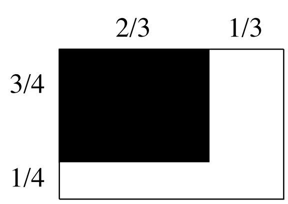
   Rickey : Lou is guilty.
   Lou : Maury is guilty.
   Maury : Ty is guilty.
   Ty : Either Rickey or Maury is guilty, but not both.

   Each innocent suspect has told the truth, each guilty suspect has lied. Name the culprits.

   (a) Lou and Ty
   (b) Maury and Ty
   (c) Rickey and Lou
   (d) Maury and Rickey
   (e) Rickey and Ty

   Solution: (a) Notice that Rickey is lying, and therefore guilty, if and only if Lou is innocent, and Lou is innocent if and only if Maury is guilty. Therefore Ty's statement cannot be true, so Ty is guilty. Therefore Maury is innocent, so Lou is guilty, so Rickey is innocent. The culprits are Lou and Ty.

5. What percentage of this rectangle is shaded?



   (a) 65%
   (b) 75%
   (c) 50%
   (d) 78%
   (e) 85%

Solution: (c)

$$\begin{aligned}\text{Total area of rectangle} &= (1/4+3/4)\cdot(2/3+1/3)\\ &= 1\cdot 1 = 1\\ \text{Shaded area} &= (2/3\cdot 3/4) = 0.5\\ \text{Fraction of Total area shaded} &= 0.5/1 = 50\%\end{aligned}$$

6. The ratio of cherry candies to grape candies in a bag is 3 : 5. After your friend eats half of the grape flavored candies and only one cherry flavored candy, the ratio of cherry to grape candies is 7 : 6. How many total candies are left in the bag?

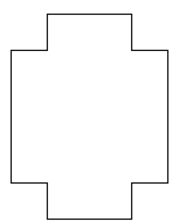
   (a) 65
   (b) 70
   (c) 68
   (d) 75
   (e) 80

   Solution: (a) Let c and g represent the number of cherry and grape candies left in the bag (after your friend eats some of them). Then we have:

   $$\frac{c+1}{2g} = \frac{3}{5} \quad \text{and} \quad \frac{c}{g} = \frac{7}{6}.$$

   These equations become: $5c + 5 = 6g$ and $6c = 7g$. Solving this system of equations we get $c = 35$ and $g = 30$, so there are $35 + 30 = 65$ candies left in the bag.

7. A rectangular sheet of wood has four small squares removed. It is then cut to make a rectangular box with open top that has a 5cm by 4cm bottom, with a volume of 60 cm^3. What is the original area of the sheet of wood?



   (a) 100 cm^2
   (b) 1004 cm^2
   (c) 104 cm^2
   (d) 110 cm^2
   (e) 210 cm^2

   Solution: (d) We know that the volume of the rectangular box $= l\cdot w\cdot h = 60$, which is equivalent to $(5\cdot 4)\cdot h = 60$, thus $h = 3$. Dividing the region in the picture into 5 rectangles and finding the area of each of the rectangles we get the surface area of the rectangular box with open top, as follows:

   $$4\cdot 5 + 2(3\cdot 5) + 2(3\cdot 4) = 20 + 30 + 24 = 74$$

Then adding the areas of the four removed squares we get the area of the rectangular sheet of wood:

$$74 + 4(3 \cdot 3) = 74 + 36 = 110 \text{ cm}^2.$$

8. The measure of an angle is 3 times the measure of the angle it is supplementary to. Find the measure of the angle in degrees.

   (a) 93°
   (b) 270°
   (c) 135°
   (d) 42°
   (e) 540°

   Solution: (c) Denote by x the measure of the angle. Then $x = 3(180 - x) = 540 - 3x$ which implies that $4x = 540$, or $x = 135°$.

9. Avogadro's number is about $6 \cdot 10^{23}$, which is 1 mole of particles. For example, there is about a mole of water molecules in one teaspoon of water. The distance to the sun from earth is approximately 150,000,000 km. If a tennis ball is approximately 6 cm in diameter, how many times could 1 mole of tennis balls, placed end-to-end, go to the sun from earth?

   (a) 2.4 times
   (b) 24,000,000,000,000 times
   (c) 2,400,000,000,000 times
   (d) 24,000,000,000,000,000,000,000,000,000,000 times
   (e) 1 time

   Solution: (c) $(6 \text{ cm}) \cdot (6 \cdot 10^{23}) = (3.6) \cdot 10^{24}$ cm.
   But 1 km $= 1000$ m $= 1000 \cdot 100$ cm $= 10^5$ cm, so $(1.5) \cdot 10^7$ km $= (1.5) \cdot 10^{12}$ cm.
   Then $(3.6) \cdot 10^{24} \text{ cm} \div (1.5) \cdot 10^{12} \text{ cm} = (2.4) \cdot 10^{12} = 2,400,000,000,000$ times.

10. A boy ate 100 cookies in five days. Each day he ate 6 more than the day before. How many cookies did he eat on the first day?

    (a) 5 cookies
    (b) 6 cookies
    (c) 7 cookies
    (d) 8 cookies
    (e) 9 cookies

Solution: (d) Let x be the number of cookies eaten on the first day.

Day	Cookies Eaten
1	x
2	$x + 1 \cdot (6)$
3	$x + 2 \cdot (6)$
4	$x + 3 \cdot (6)$
5	$x + 4 \cdot (6)$

TOTAL $5x + 10(6) = 100$ cookies

Solve for x:

$$\begin{aligned} 5x + 10 \cdot (6) &= 100 \\ 5x + 60 &= 100 \\ 5x &= 40 \\ x &= 8. \end{aligned}$$