First Annual High School Problem Solving Contest

Department of Mathematics

Name:

Grade Level:

High School Name:

Problem	1	2	3	4	5	6	**Total**
Points	10	10	10	10	10	10	60
Score							

Note:

- There are 6 problems.
- Clearly show all the steps in your solution to earn credit.
- You have 2 hours to solve the problems.

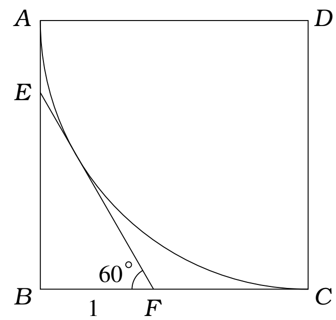
Good Luck!!

Problem 1 (10 points)

A rising number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. When all five-digit rising numbers are arranged from smallest to largest, find the 100th number in the list.

Problem 2 (10 points)

A circular arc is drawn in a square with center D at one of the vertices of the square and the arc is tangent to the opposite two sides of the square. The arc is also tangent to the hypotenuse of the $30° - 60° - 90°$ triangle BFE as shown, where $BF = 1$. What is the radius of the circle?



Problem 3 (10 points)

A number A has 666 threes as its digits and a number B has 666 sixes as its digits. What are the digits in the product $A \times B$?

Problem 4 (10 points)

Suppose the numbers $a_1, a_2, \ldots, a_{100}$ satisfy:

$$\begin{aligned}
&a_1 - 4a_2 + 3a_3 \geq 0 \\
&a_2 - 4a_3 + 3a_4 \geq 0 \\
&\vdots \\
&a_{98} - 4a_{99} + 3a_{100} \geq 0 \\
&a_{99} - 4a_{100} + 3a_1 \geq 0 \\
&a_{100} - 4a_1 + 3a_2 \geq 0
\end{aligned}$$

Let $a_1 = 1$. Find the values of $a_2, a_3, \ldots, a_{100}$.

Problem 5 (10 points)

Prove that, if a polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

with integer coefficients has odd values at $x = 0$ and $x = 1$, then the equation

$$P(x) = 0$$

has no integer roots.

Problem 6 (10 points)
Is there a triangle, whose heights have lengths 1, $\sqrt{5}$, $1+\sqrt{5}$?