The Subject Math GRE Test: Linear Algebra

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GRE

- The Subject Math GRE is, roughly, partitioned as follows:

  (a) Calculus and its applications ($\sim$ 50%)
  (b) Linear Algebra, Abstract Algebra, Number Theory. ($\sim$ 25%)
  (c) Discrete Math, Differential stuff, Topology, Probability, Statistics, etc. ($\sim$ 25%).

- So, you should expect $10 - 15\%$ of the questions in the test to be on Linear Algebra topics. That is $7 - 10$ questions.

- In this presentation, we will go over Linear Algebra concepts and sample problems[1].

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[1] I found these problems on the internet, I do not claim that I created them.

Question 1

Suppose T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$ such that

$$T\begin{pmatrix} 1 \\ -3 \end{pmatrix} = 5 \qquad \text{and} \qquad T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = -2$$

Then,

$$T\begin{pmatrix} 0 \\ 2 \end{pmatrix} =$$

(A) -4 **(B)** $-\frac{7}{2}$ **(C)** -1 **(D)** 3 **(E)** $\frac{9}{22}$

Concepts needed: Linear combinations, basis, linear transformation defined in a basis.

Difficulty: Easy.

Question 2

Suppose $\mathcal{P}_7$ is the set of polynomials with coefficients in $\mathbb{Z}_5$ and degree less than or equal to 7 (union $\{0\}$). If the operator D sends $p(x)$ in $\mathcal{P}_7$ to its derivative $p'(x)$, what are the dimensions of the null space n and range r of D?

(A) $n = 1, r = 6$ **(B)** $n = 1, r = 7$ **(C)** $n = 2, r = 5$

(D) $n = 2, r = 6$ **(E)** $n = 3, r = 5$

Concepts needed: Null space/Kernel, operator/linear transformation, dimension formula/first isomorphism theorem.

Difficulty: Medium (only because of language).

Question 3

Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, then $A^{500} =$

(A) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ **(B)** $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ **(C)** $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ **(E)** $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Concepts needed: Permutation matrix, change of basis matrix.

Difficulty: Medium.

Question 4

Suppose V is a real vector space of finite dimension n. Call the set of matrices from V into itself $\mathcal{M}(V)$. Let $T \in \mathcal{M}(V)$. Consider the two subspaces

$$\mathcal{U} = \{X \in \mathcal{M}(V);\ TX = XT\} \qquad \mathcal{W} = \{TX - XT;\ X \in \mathcal{M}(V)\}$$

Which of the following must be TRUE?
I. If V has a basis containing only eigenvectors of T then $\mathcal{U} = \mathcal{M}(V)$.
II. $dim(\mathcal{U}) + dim(\mathcal{W}) = n^2$.
III. $dim(\mathcal{U}) < n$.

(A) I only **(B)** II only **(C)** III only

(D) I and II only **(E)** I, II, and III

Concepts needed: Image and kernel of a linear transformation, eigenvectors, first isomorphism theorem/dimension formula.
Difficulty: Hard.

Question 5

Suppose A is a 3×3 matrix such that

$$det(A - \lambda I) = -\lambda^3 + 3\lambda^2 + \lambda - 3$$

where I is the 3×3 identity matrix. Which of the following are TRUE?
I. The trace of A is 3.
II. The determinant of A is -3.
III. The matrix A has eigenvalues -3 and 1.

(A) I only **(B)** II only **(C)** III only

(D) I and II only **(E)** I, II, and III

Concepts needed: Characteristic polynomial of a matrix, eigenvalues, determinant and trace.

Difficulty: Medium/Hard (because of obscurity of concepts needed).

Question 6

Suppose A and B are $n \times n$ matrices with real entries. Which of the following are TRUE?
I. The trace of A^2 is non-negative.
II. If $A^2 = A$, then the trace of A is non-negative.
III. The trace of AB is the product of the traces of A and B.

(A) II only **(B)** III only **(C)** I and III only

(D) II and III only **(E)** I, II, and III

Concepts needed: Trace.

Difficulty: Hard (unless you look at the choices)

Thanks!